ECON 2127/8014
Tutorial questions – Week 1
Semester 2, 2009

1. Let $\mathbb{N}_+ := \{1, 2, 3, \ldots\}$. Use induction to prove that for all $n \in \mathbb{N}_+$,

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. Let $g(x) = 3x^4 + 2x^3 + x - 1$ and $f(x) = x^2 + 1$. Use polynomial long division to find the quotient and remainder of the division of g by f. (If you don't remember polynomial long division, look it up somewhere.)

3. Prove that every polynomial with odd degree and real coefficients has a real root. (You can use any result discussed in class.)

4. (Triangle Inequality) Prove that for all $x, y \in \mathbb{R}$,

$$|x + y| \leq |x| + |y|.$$

5. Consider two sequences (a_n) and (b_n), defined by

$$a_n := \frac{(-1)^n}{n}, \quad b_n := \frac{n}{n^2 + 1}.$$

(i) Determine the limits of (a_n) and (b_n), and provide formal proofs for your claims, using the definition of convergence.

(ii) Determine whether the sequences are monotone, and find a monotone subsequence for each sequence.

6. Let (s_n) be a sequence in $\mathbb{R}$. Prove that $\lim s_n = 0$ *iff* $\lim |s_n| = 0$.

7. If (s_n) is a convergent sequence whose elements all belong to a closed interval $[a, b]$, prove that its limit also lies in $[a, b]$.