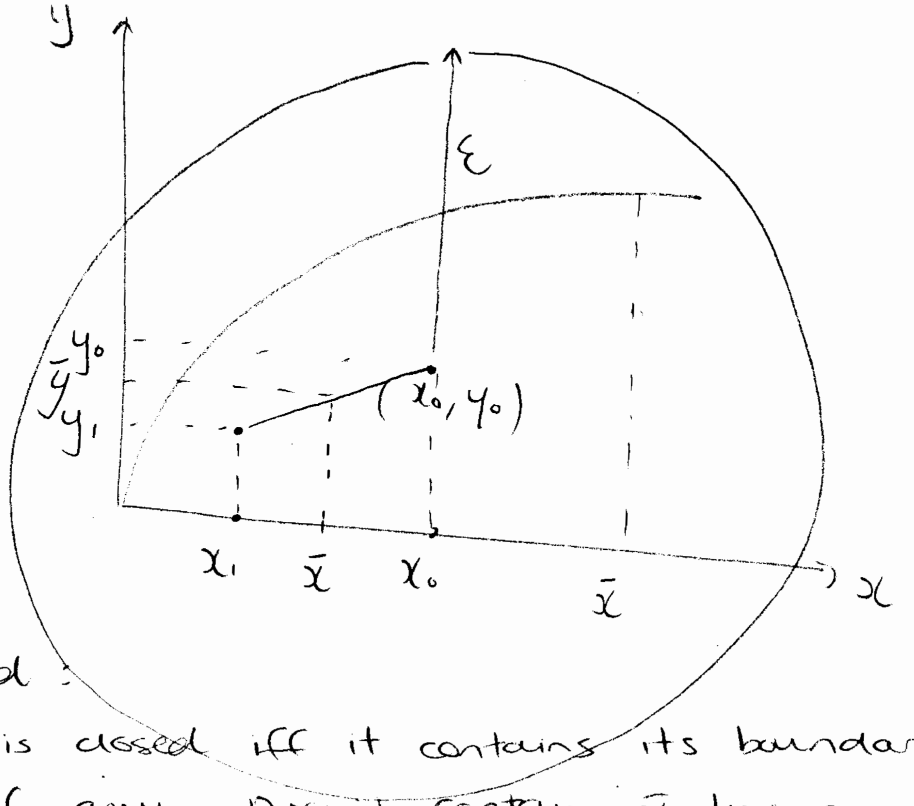
1. $Y = \{(x,y) \in \mathbb{R}^2_{++} : y \leq \sqrt{x}\}$



Closed:

A set is closed iff it contains its boundary points, if any. Doesn't contain $\bar{x}$ line so NO.

Open: A set is open if $\forall x \in X \exists \varepsilon > 0$ s.t $N_\varepsilon(x) \subset X$

Not so for points on production fn so NO

Bounded if $\forall x_0, y_0 \in Y \exists \varepsilon < 0$ s.t $X \subset N_\varepsilon(x_0, y_0)$
: YES (we can bound it (as illustrated))

Convex if $\forall x, x', y, y' \in Y$, the convex combo
$\bar{x} = \lambda x + (1-\lambda)x'$ (or $\bar{y} = \lambda y + (1-\lambda)y'$) is in Y. Illustrated for $\bar{x}, \bar{y}$ & $\lambda = \frac{1}{2}$.
$\Downarrow$
$\exists \lambda \in [0,1]$

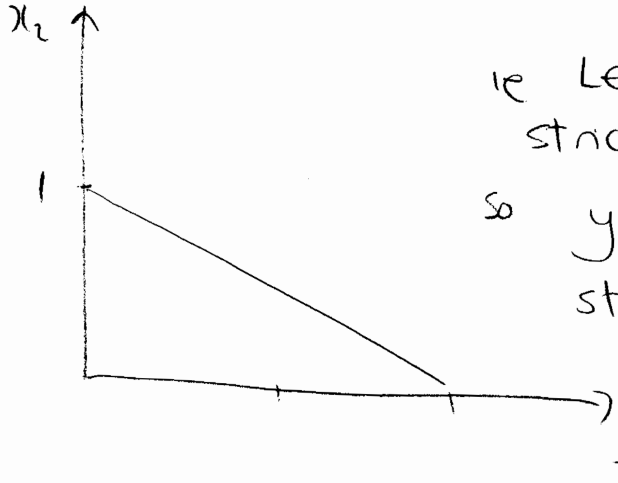
Strictly convex if $\forall \lambda \in (0,1)$ ie not including 0 or 1, $\bar{x}, \bar{y}$ is in the interior of Y. NO - look at $\forall \lambda$ the convex combo x_0, x_1. lies on boundary of set.

2.

a. $y = x_1 + 2x_2$

level set $x_1 + 2x_2 = 2 \quad (\bar{y})$

ie $x_2 = 1 - \frac{1}{2} x_1$



ie Level sets NOT strictly concave/convex

So y is neither strictly q-concave or strictly q convex.

c. $y = (x_1^2 + x_2^2)^{1/2}$

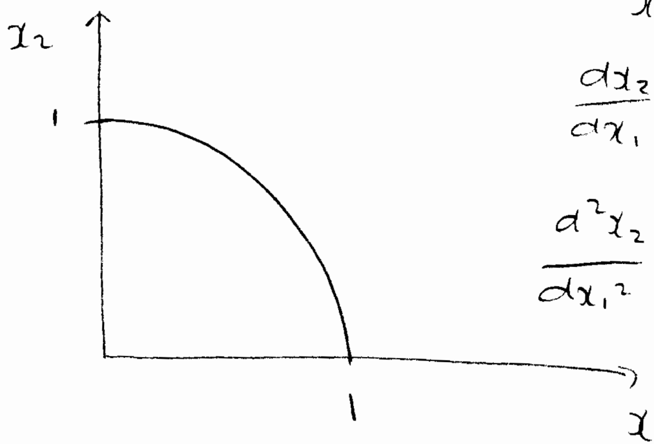
Level set $(x_1^2 + x_2^2)^{1/2} = 1$

Square both sides $\Rightarrow x_2^2 + x_1^2 = 1$

$\Rightarrow$ Circle $\bar{c}$ radius 1

ie $x_2^2 \quad 1 - x_1^2$

$x_2 = (1 - x_1^2)^{1/2}$



$\frac{dx_2}{dx_1} = \frac{1}{2} \cdot -2x_1 (1 - x_1^2)^{-1/2}$

$< 0 \ \forall x_1$

$\frac{d^2 x_2}{dx_1^2} = -(1 - x_1^2)^{-1/2} +$

$\frac{1}{4} \cdot 2x_1 (1 - x_1^2)^{-3/2} \times -2x_1$

$< 0 \ \forall x_1$

$\therefore \downarrow$ sloping & s. concave.

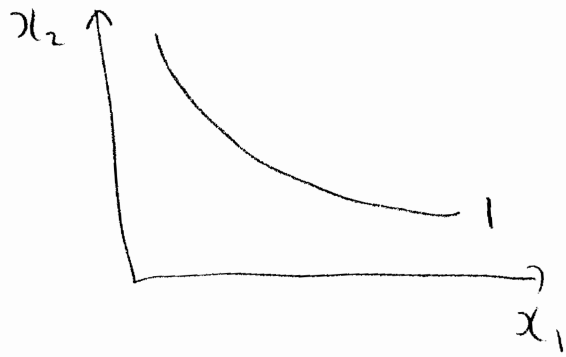
$\therefore y$ is strictly quasi convex.

b. $y = x_1^2 x_2^2$

Level set $\sqrt{x_2^2} = \sqrt{\frac{1}{x_1^2}}$

$$x_2 = \frac{1}{x_1}$$

$\Rightarrow$



Level sets are strictly convex

So y is strictly quasi concave.

3. $\pi = TR - TC$

$TC = aQ + bQ^2$

$mc = a + 2bQ$

$p = L - nQ \Rightarrow TR = LQ - nQ^2$

$$\pi = -(b+n)Q^2 + (L-a)Q$$

$$\frac{d\pi}{dQ} = -2(b+n)Q + (L-a) = 0$$

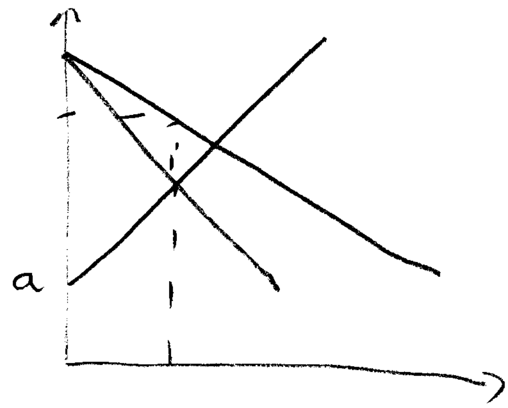
$$\Rightarrow Q = \frac{L-a}{2(b+n)}$$

$\therefore P =$

SOC $\dfrac{d\pi^2}{dQ^2} = -2(b+n) < 0$

$\therefore \pi$ max

Sketch :



$$Q_m = \frac{L-a}{2(b+n)}$$

Now government introduces tax of t per unit of quantity, increasing TC by tQ.

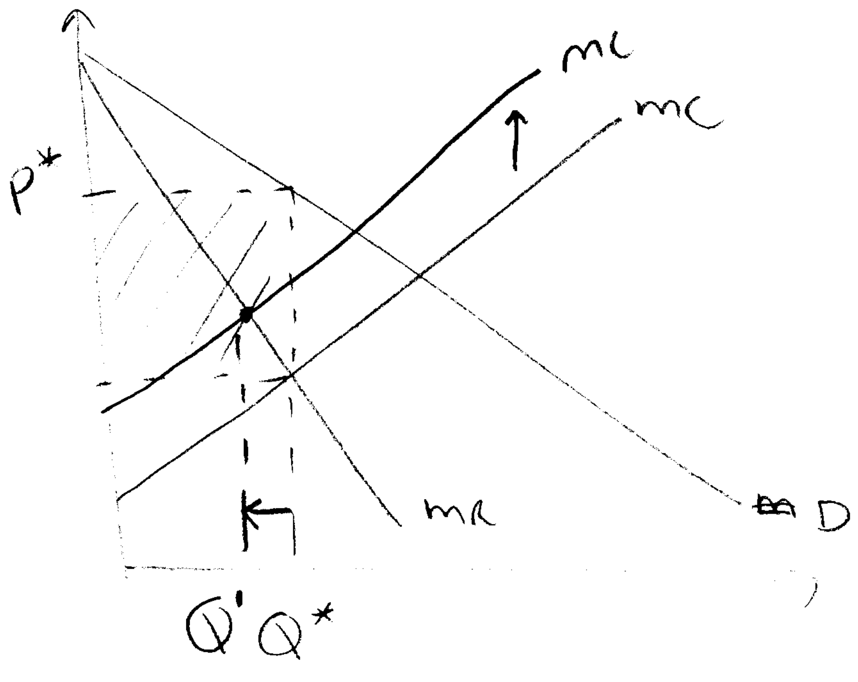
$$\Pi = -(b+n)Q^2 + (L-a)Q - tQ$$

$$= -(b+n)Q^2 + (L-a-t)Q$$

$$\frac{d\Pi}{dQ} = 0 \Rightarrow +2(b+n)Q = L-a-t$$

$$Q = \frac{L-a-t}{2(b+n)}$$

$$\frac{dQ}{dt} = -\frac{1}{2(b+n)}$$



Tax shifts mc ↑, Q ↓

4.

$$|A| = -(1-x) + (1-x)[(5-x)(1-x)-4]$$

$$= -(1-x) + (1-x)[5-6x+x^2-4)]$$

$$= -(1-x) + (1-x)(x^2-6x+1)$$

$$= (1-x)[x^2-6x+1-1]$$

$$= (1-x)(x^2-6x)$$

$$= x(1-x)(x-6)$$

If $|A| \neq 0$ $R(A) = 3$

ie If $x \neq 0, 1, 6$ then rank is 3

Because $\begin{vmatrix} 5-x & 2 \\ 1 & 0 \end{vmatrix} = -2 \neq 0$ the rank of A is always 2 when $x = 0, 1, 6$.

or $|A| = (5-x)(1-x)^2 - 4(1-x) - 1.(1-x)$

$$= (1-x)[(5-x)(1-x)-5)]$$

$$= (1-x)[\cancel{5}+x^2-6x-\cancel{5}]$$

$$= (1-x)x(x-6)$$

✓

5. $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

Characteristic equation $\Rightarrow |A - \lambda I|$

$(A - \lambda I) q = 0$

$$\Rightarrow \left(\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2$$

$$= 6 + \lambda^2 - 5\lambda - 2$$

$$= \lambda^2 - 5\lambda + 4$$

$$= (\lambda - 4)(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 4, 1.$$

For $\lambda = 4$:

$$\begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -2q_1 + 2q_2 = 0$$

$$\Rightarrow q_1 = q_2$$

combine $\bar{c}$ $q_1^2 + q_2^2 = 1 \Rightarrow 2q_2^2 = 1$

$$q_2^{x} = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For $\lambda = 1 \Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow q_1 = -2q_2$$

$$q \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

$Q^{-1}AQ$: Q <u>NOT</u> orthogonal so $Q^{-1} \neq Q^T$

$$Q^{-1}AQ = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 4 & 8 \\ -1 & 1 \end{pmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 12 & 0 \\ 0 & 3 \end{pmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \Lambda$$

c. Since both eigenvalues are positive, A is positive definite.

6. a. $dq = f'(p^c)\,dp^c \Rightarrow dq - f'(\,)dp^c = 0$

$dq = g'(p^p)\,dp^p \Rightarrow dq - g'(\,)dp^p = 0$

$dp^p = dp^c + ds \qquad dp^p - dp^c = ds$

b. $$\begin{bmatrix} 1 & -f' & 0 \\ 1 & 0 & -g' \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} q \\ p^c \\ p^p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ ds \end{bmatrix}$$

$$dq = \begin{vmatrix} 0 & -f' & 0 \\ 0 & 0 & -g' \\ ds & 1 & 1 \end{vmatrix} / |A| = \frac{ds\,(f'g')}{-g' + f'}$$

$$\text{ie } \frac{dq}{ds} = \frac{f'g'}{f'-g'} > 0$$

$$dp^c = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -g' \\ 0 & ds & 1 \end{vmatrix} / |A| = \frac{g'\,ds}{f'-g'}$$

$$\Rightarrow \frac{dp^c}{ds} = \frac{g'}{f'-g'} < 0$$

$$dp^p = \begin{vmatrix} 1 & -f' & 0 \\ 1 & 0 & 0 \\ 0 & -1 & ds \end{vmatrix} / |A| = \frac{ds \cdot f'}{f'-g'}$$

$$\Rightarrow \frac{dp^p}{ds} = \frac{f'}{f'-g'} > 0$$

c. In equilibrium $f(p^c) = g(p^p) = g(p^c+s)$

$\Rightarrow F(p^c,t) = f(p^c) - g(p^c+s) = 0$

Take TDs $\rightarrow f'(\cdot)dp^c - g'(\cdot)dp^c - g'(\,)ds = 0$

$$\therefore \quad dp^c = \frac{g'(\cdot)\,ds}{f'-g'}$$ as in b.

From a) $dp^p = dp^c + ds$

$$= \frac{g'}{f'-g'} \cdot ds + ds \cdot \frac{f'-g'}{f'-g'}$$

$$= \frac{f'}{f'-g'} \cdot ds$$ as in b.

And $dq = f'(p^c)\,dp^c = \frac{f' \cdot g'}{f'-g'} \cdot ds$

as in b.

7. $U = x_1 x_2 x_3$

$$dU = U_1 dx_1 + U_2 dx_2 + U_3 dx_3$$

$$= x_2 x_3 dx_1 + x_1 x_3 dx_2 + x_1 x_2 dx_3$$

$= 0$ $\bar{c}$ $dx_1 = 0$

$$\Rightarrow \quad x_1 x_3 dx_2 + x_1 x_2 dx_3 = 0$$

$$MRS_{2,3} = \frac{dx_3}{dx_2} = -\frac{x_1 x_3}{x_1 x_2} = -\frac{x_3}{x_2}$$

ie Indifference curves slope down.

8. $$x_1^2 e^{3x_2 + x_1 x_3} + 2x_2^3/x_1$$

Young's Theorem says $f_{12} = f_{21}$ $f_{13} = f_{31}$ $f_{23} = f_{32}$

$$f_2 = 3x_1^2 e^{3x_2 + x_1 x_3} + \frac{6x_2^2}{x_1}$$

$$f_3 = x_1^3 e^{3x_2 + x_1 x_3}$$

$$f_{23} = 3x_1^3 e^{3x_2 + x_1 x_3}$$

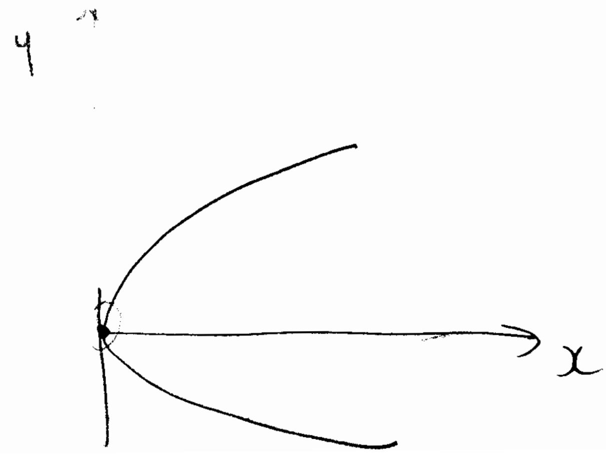
$$f_{32} = 3x_1^3 e^{3x_1 + x_1 x_3} > 0.$$

9. $$y^2 - x = 0 = F(y, x)$$

Imp. Fn rule $\Rightarrow$ $2y\,dy - dx = 0$

or $\Rightarrow \dfrac{dy}{dx} = \dfrac{-F_x}{F_y}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y}$ $y \neq 0$

ie $F_y \neq 0 \Rightarrow y \neq 0$



At all $(x,y) \neq (0,0)$ can define ε-nbd around y.