THE AUSTRALIAN NATIONAL UNIVERSITY

Mathematics for Economists A (ECON2125)
Mathematical Techniques in Economics 1 (ECON8013/ECON4021)

Mid-Semester Examination, April 2006

READING TIME: 15 minutes
WRITING TIME: 90 minutes

Permitted materials: Non-programmable calculators

This exam will be marked out of 90. The marks for each question are indicated at the end of the question. **Answer ALL questions**.

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1. A firm's production set is given by $Y = \{(x, y) : y \leq \sqrt{x},\ 0 \leq x < \overline{x}\}$. Sketch this set and explain carefully whether it is closed, open, bounded, convex and/or strictly convex.

[10 marks]

2. Sketch typical level sets for the following functions and state whether the functions are strictly quasiconcave, strictly quasiconvex or neither.

a. $y = x_1 + 2x_2$, $x_1, x_2 > 0$
b. $y = x_1^2 x_2^2$, $x_1, x_2 > 0$
c. $y = (x_1^2 + x_2^2)^{1/2}$, $x_1, x_2 > 0$

[10 marks]

3. Suppose the inverse demand function facing a monopolist is given by $P = L - nQ$, where P is the price of the good and Q is its quantity. Total costs are given by $TC = aQ + bQ^2$. Assume that L, n, a and b are all positive constants and also that $L > a$.

a. Set up the firm's profit-maximisation problem and first-order condition in order to solve for the optimal level of output, Q^*. Carefully illustrate your answer using the demand, marginal revenue and marginal cost curves.
b. Is the second order sufficient condition satisfied for a global maximum?
c. Suppose that the government introduces a tax of t per unit, which increases total costs by tQ. How does the optimal level of Q vary with t? Illustrate your answer.

[15 marks]

4. Determine the rank of $\mathbf{A} = \begin{bmatrix} 5-x & 2 & 1 \\ 2 & 1-x & 0 \\ 1 & 0 & 1-x \end{bmatrix}$ for all values of x.

[10 marks]

5. Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ and the eigenvalue problem $\mathbf{A}\boldsymbol{q} = \lambda\boldsymbol{q}$.

a. Write the characteristic equation and find the eigenvalues of **A**.
b. Find *any* corresponding eigenvectors and form the matrix **Q.** Use this to diagonalise **A** (noting that **A** is not symmetric)**.**
c. Based on the above, what can you say about the definiteness of **A**?

[15 marks]

6. Consider the following supply and demand model:

$$q_d = f(p^c), \quad f'(p^c) < 0 \qquad (1)$$
$$q_s = g(p^p), \quad g'(p^p) > 0 \qquad (2)$$
$$p^p = p^c + s \qquad (3)$$

where q_d is quantity demanded, q_s is quantity supplied, p^c is the price that consumers pay for the product, p^p is the price that producers receive and s is a per unit subsidy. In equilibrium $q_d = q_s = q$, which is determined endogenously along with p^c and p^p.

a. Take total differentials of the first three equations above.
b. Use Cramer's Rule to find the effect on p^c, p^p and q from an increase in s of ds.
c. Express this model in the form of a single equation, $F(p^c, s) = 0$ and take total differentials to confirm your answers for part b.

[15 marks]

7. Take the total differential of the utility function $U = x_1 x_2 x_3$ and use this to show that the indifference curves in (x_2, x_3)-space (i.e. the plane with x_1 held fixed) are downward sloping.

[5 marks]

8. Illustrate Young's Theorem by showing that $f_{23} = f_{32}$ for the three-input production function $f(x_1, x_2, x_3) = x_1^2 e^{3x_2 + x_1 x_3} + 2x_2^3 / x_1$.

[5 marks]

9. Consider the implicit function $y^2 - x = 0$. Sketch a graph and use the implicit function rule to find dy/dx where it is defined. At which points (x, y) will it be possible to define y as a function of x within an epsilon neighbourhood of the points?

[5 marks]