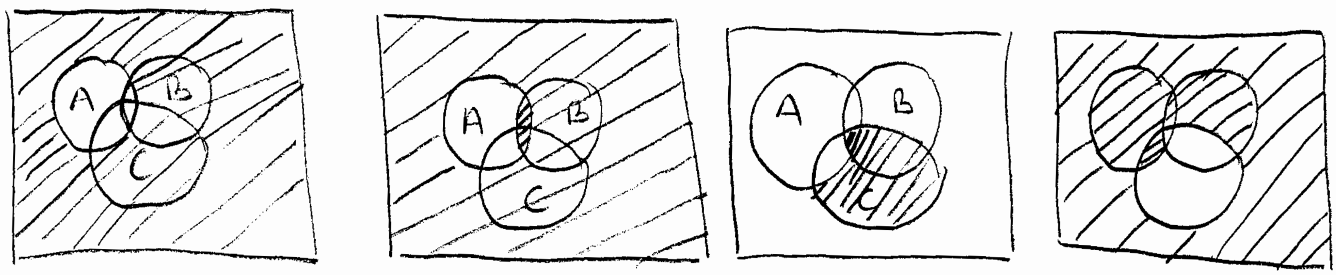
MID-SEMESTER EXAM, 2003

1. $\left(\overline{(\bar{A} \cup B) \cap C}\right)$



$\bar{A}$ $\qquad$ $\bar{A} \cup B$ $\qquad$ $(\bar{A} \cup B) \cap C$ $\qquad$ $\overline{(\bar{A} \cup B) \cap C}$

2. From 1st principles

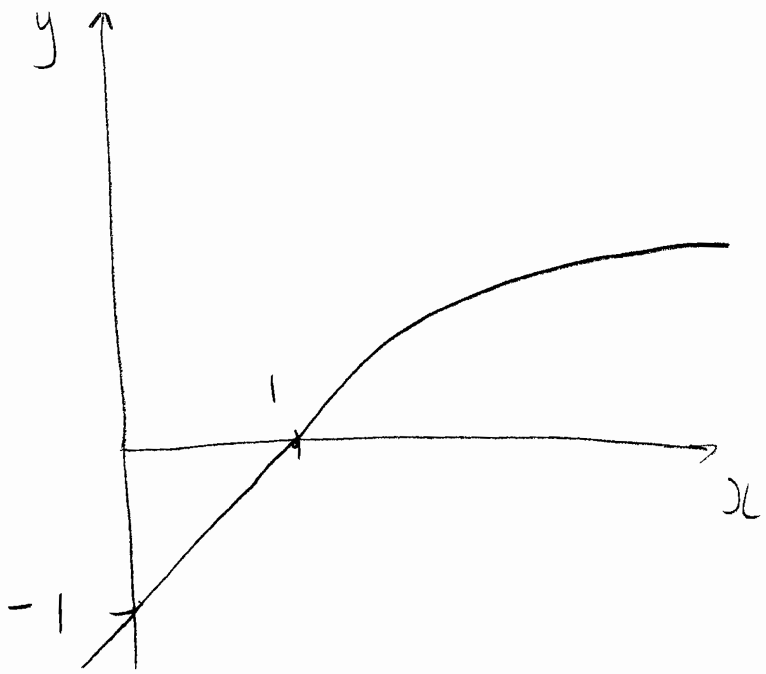
$$f(x) = 4x^2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{4(x+\Delta x)^2 - 4x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{4x^2} + 8x\cancel{\Delta x} + 4(\Delta x)^{\cancel{2}} - \cancel{4x^2}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 4(2x + \Delta x) = 8x$$

3. $y = \begin{cases} x-1, & x \leq 1 \\ \ln x, & x > 1 \end{cases}$



$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 0$

$\therefore \lim_{x \to 1} f(x) = 0$ & the function is continuous

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^-} f'(x)$

$1 \qquad \frac{1}{x} = 1$ at $x = 1$

$\therefore$ the function is differentiable.

4. a.

$$\begin{bmatrix} 6 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b \\ 1 \\ 1 \end{bmatrix}$$

$$A \qquad \underset{\sim}{x} = \underset{\sim}{d}$$

$$|A| = 6\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 6(-1) + 2(3) = 0$$

$\therefore$ No unique solution.

$$\begin{vmatrix} 6 & 0 \\ 2 & 1 \end{vmatrix} \neq 0 \quad \therefore \quad r(A) = 2$$

b. $B = [A : d]$:

$$\left[\begin{array}{ccc|c} 6 & 0 & 2 & b \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right]$$

<u>If</u> $r(B) = r(A)$ there are ∞ solutions. For this we need ALL 3×3 sub-matrices to have determinants $= 0$

ie If
$$\begin{vmatrix} 0 & 2 & b \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 0(0) - 2(1-2) + b(1-2) = 0$$
$$\Rightarrow 2 - b = 0 \Rightarrow b = 2$$

$$\begin{vmatrix} 6 & 0 & b \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 6(1-2) + b(4-1) = -6 + 3b$$
$$= 0 \text{ if } b = 2$$

$$\begin{vmatrix} 6 & 2 & b \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 6(0) - 2(2-1) + b(2-1)$$
$$= 0 \text{ if } b = 2$$

ie. If $b = 2 \Rightarrow r(B) = r(A) \Rightarrow \infty$ solutions

If $b \neq 2 \Rightarrow r(B) > r(A) \Rightarrow$ inconsistent equations

5. $y = f(L) \quad f'(L) > 0 \quad f''(L) < 0$

a. $\pi = p(1-t)f(L) - wL$

FONC: $\dfrac{d\pi}{dL} = p(1-t)f'(L) - w = 0$

ie. $VMPL = MC_L$

value of marg. product of L & marginal cost of L.

b. SOSC: $\dfrac{d^2\pi}{dL^2} = p(1-t)f''(L) < 0 \quad \forall L$

$\therefore$ global

c. Comp Stats – $\uparrow p$

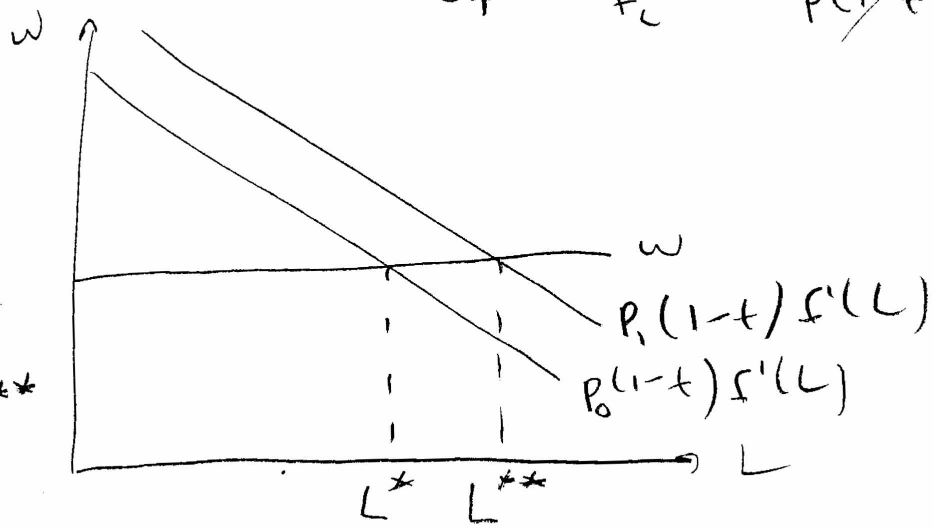
Method 1: FONC: $p(1-t)f'(L^*(p)) - w = 0$

$= F(p, t, w, L)$

$$\frac{d\,FONC}{dp} = (1-\cancel{t})f'(L) + p(1-\cancel{t})f''(L)\cdot\frac{dL^*}{dp} = 0$$

$$\Rightarrow \frac{dL^*}{dp} = -\frac{\overset{\oplus}{f'(L)}}{\underset{\ominus}{pf''(L)}} > 0$$

or: Implicit fn. rule $\dfrac{dL^*}{dp} = -\dfrac{F_p}{F_L} = \dfrac{-(1-\cancel{t})f'(L)}{p(1-\cancel{t})f''(L)} > 0$



$\uparrow p \Rightarrow$
$p(1-t)f'(L)$
shifts right
$\Rightarrow \uparrow L^*$ to L^{**}

Comp stat for $\uparrow t$

$$\frac{dL^*}{dt} = \frac{-F_t}{F_L} = \frac{+\cancel{p} f'(L)}{\cancel{p}(1-t) f''(L)} < 0$$

(opposite shift in previous diagram)

6. IS : $Y = C + I + G + X$

$$= 15 + 0.8(Y - (-25 + 0.25Y)) + 65 - r$$
$$+ 94 + 40 - 0.1Y$$

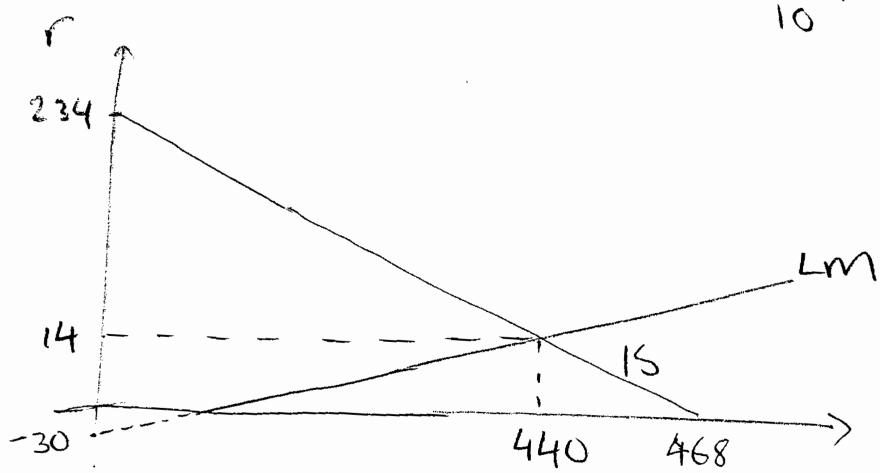
$$\Rightarrow \quad Y = 234 + 0.6Y - r - 0.1Y$$

$$\Rightarrow (1) \quad Y = 468 - 2r \quad \text{or} \quad r = 234 - \tfrac{1}{2}Y$$

LM: $m^d = m^s \qquad L = 5Y - 50r = 500 \; (=m)$

$$\Rightarrow \quad 5Y = 1{,}500 + 50r \quad (2)$$

$$Y = 300 + 10r \quad \text{or} \quad r = \tfrac{1}{10}Y - 30$$



In matrix form (1) & (2) $\rightarrow \begin{bmatrix} 1 & 2 \\ 1 & -10 \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} 468 \\ 300 \end{bmatrix}$

$$Y^* = \begin{vmatrix} 468 & 2 \\ 300 & -10 \end{vmatrix} \Big/ |A| = \frac{-4680 - 600}{-12} = 440$$

$$r^* = \begin{vmatrix} 1 & 468 \\ 1 & 300 \end{vmatrix} \Big/ |A| = \frac{300 - 468}{-12} = 14 \; (\%)$$

7. a. $U(x_1, x_2) = x_1^{a} x_2^{b}$

$$dU = U_1\,dx_1 + U_2\,dx_2 = 0$$

$$\Rightarrow \left.\frac{dx_2}{dx_1}\right|_{dU=0} = -\frac{U_1}{U_2} = \frac{-a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}}$$

$$= \frac{-a x_2}{b x_1}$$

$$\frac{d^2 x_2}{dx_1^2} = \frac{\partial}{\partial x_1}\left(\frac{-a x_2}{b x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{-a x_2}{b x_1}\right) \cdot \frac{dx_2}{dx_1}$$

$$= \frac{a}{b} x_2 x_1^{-2} - \frac{a}{b x_1} \cdot \frac{-a x_2}{b x_1}$$

$$= \frac{a}{b}\frac{x_2}{x_1^2} + \frac{a^2}{b^2}\frac{x_2}{x_1^2} > 0$$

∴ indifference curves are strictly convex.

b. $$\left.\frac{dx_2}{dx_1}\right|_{dU=0} = \frac{-U_1}{U_2} = \frac{\cancel{+\frac{1}{r} A[\cdot]^{-\frac{1}{r}-1}} \times -r\delta x_1^{-r-1}}{\cancel{-\frac{1}{r} A[\cdot]^{-1/r-1}} \times -r(1-\delta) x_2^{-r-1}}$$

$$= -\frac{\delta}{1-\delta} \cdot \frac{x_2^{r+1}}{x_1^{r+1}}$$