

AUSTRALIAN NATIONAL UNIVERSITY

**MATHEMATICS FOR ECONOMISTS A (ECON2125)
MATHEMATICAL TECHNIQUES IN ECONOMICS 1
(ECON8013/ECON4021/ECDV8113)**

Mid-Semester Examination, May 2003

Study Period: 15 Minutes

Time Allowed: 90 minutes

Permitted Materials: Nonprogrammable Calculators

Answer ALL questions. This exam is marked out of 80. Marks for individual questions are given at the end of each question.

1. Use as many Venn diagrams as you need to illustrate *clearly* the following expression, assuming that no pair of sets A, B and C are disjoint and where none of these sets is a subset of either of the other two.

$$\left(\overline{A \cup B} \cap C\right)$$

[5 marks]

2. From first principles, i.e., using the definition

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

show that if $f(x) = 4x^2$ then $f'(x) = 8x$.

[5 marks]

3. Sketch the following function and state whether it has a limit, whether it is continuous and whether it is differentiable at $x = 1$

$$y = \begin{cases} x - 1, & x \leq 1 \\ \ln x, & x > 1 \end{cases}$$

[6 marks]

4. Consider the system of equations:

$$6x_1 + 2x_3 = b$$

$$2x_1 + x_2 + x_3 = 1$$

$$x_1 + 2x_2 + x_3 = 1$$

where b is some real number.

- a. Express this system of equations in the form $Ax = d$ where A is a 3×3 matrix and x is the vector of endogenous variables. Will there be a unique solution? What is the rank of A ?
- b. Form the matrix $B = [A:d]$. For what value(s) of b will there be infinitely many solutions? For what values of b will the equations be inconsistent? Explain.

[12 marks]

5. Suppose that a firm produces output, Y , using labour, L , as its only factor of production. The production function is

$$Y = f(L)$$

where $f'(L) > 0$ and $f''(L) < 0$. Labour must be paid a constant wage, w , and output is sold at a constant price, p . In addition, the firm pays:

- a constant rate, t , of sales tax so that the firm receives $p(1-t)$ per unit output.

The firm chooses its labour input to maximise profits net of all taxes.

- a. Write down the first-order condition for the firm's optimisation problem. Interpret this condition.
- b. Is the second-order condition satisfied for a *local* maximum level of L ? For a *global* maximum? Explain.
- c. Find the comparative static effects on the optimal choice of labour input of changes in p . Does an increase in the market price increase or decrease labour input. Illustrate the results diagrammatically. Repeat your analysis for changes in t .

[20 marks]

6. Consider the open economy IS-LM model described by

$$C = 15 + 0.8(Y - T)$$

$$T = -25 + 0.25Y$$

$$I = 65 - r$$

$$G = 94$$

$$X = 40 - 0.1Y$$

$$L = 5Y - 50r$$

$$M = 1,500$$

where C is consumer expenditure, T is tax revenue, Y is aggregate output, I is investment expenditure, r is the interest rate, G is government expenditure, and X is net exports (exports *minus* imports). L is money demand and M is the fixed money supply.

- a. Noting that $Y = C + I + G + X$ find an expression for the IS curve. Sketch this curve in the r - Y plane.
- b. Find an expression for the LM curve and sketch it on the same diagram.
- c. Convert the IS and LM equations into matrix form and solve for the optimal levels of Y and r .

[20 marks]

7. Take total differentials in order to find $\left. \frac{dx_2}{dx_1} \right|_{dU=0}$ for the following utility functions.

a. $U(x_1, x_2) = x_1^a x_2^b$, $a, b > 0$,

b. $U = A[\delta x_1^{-r} + (1-\delta)x_2^{-r}]^{-1/r}$, $A > 0$, $r > -1$, $0 < \delta < 1$

In the case of part a, and noting that we cannot take x_2 as constant, find $\left. \frac{d^2 x_2}{dx_1^2} \right|_{dU=0}$ in order to show that the indifference curves are strictly convex to the origin.

[12 marks]
