THE AUSTRALIAN NATIONAL UNIVERSITY

First Semester Examination (2006)

MATHEMATICAL TECHNIQUES IN ECONOMICS 1 AND MATHEMATICS FOR ECONOMICS A

(ECON 8013 / ECON 4021/ ECON 2125)

Writing period : 3 Hours duration
Study period : 30 Minutes duration
Permitted materials : Non-programmable Calculators

All questions should be completed in the script book provided.

There are NINE questions in total. Answer ALL questions. This exam is marked out of 120. Marks for individual questions are given at the end of each question.

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1. Show that $y = f(x_1, x_2) = (x_1^{-2} + x_2^{-2})^{-1/2}$ is homogenous of degree one and that $f_1(x_1, x_2)$ is homogeneous of degree zero.

[10 marks]

2. Which of the following functions are homothetic?

a. $y = f(x_1, x_2) = x_1 x_2$

b. $y = f(x_1, x_2) = x_1 x_2 + x_2$

c. $y = f(x_1, x_2) = (x_1 x_2)^2 + 1$

[10 marks]

3. Consider an individual with utility function $U(x_1, x_2) = (H - x_1)x_2$, where x_1 is the number of hours worked per period, x_2 is the quantity of a consumption good and H is total hours available per period.

a. Find the slope of an indifference curve $dx_2/dx_1\big|_{\bar{U}}$ and the curvature of the indifference curve $d^2x_2/dx_1^2\big|_{\bar{U}}$. Are the indifference curves either strictly concave or strictly convex? Sketch an indifference curve and use your diagram to argue that utility is strictly quasiconcave.

b. Confirm that utility is strictly quasiconcave using a bordered Hessian matrix.

[10 marks]

4. Consider a closed-economy macroeconomic model characterised by the following equations:

$$C = a + b(1-T)Y, \quad a > 0, 0 < b < 1, 0 < T < 1$$
$$I = e - fr, \quad e, f > 0$$
$$G = G_0$$
$$M_d = kY - hr, \quad k, h > 0$$
$$M_s = \frac{M_0}{P_0}$$

where C is consumption, T is the tax rate, I is investment, r is the interest rate, G is government spending, M is nominal money supply, and P is the price level. In this model, a,b,e,f,k and h are fixed parameters, G, T, M and P are exogenous and Y and r are endogenous.

a. Use the equations above to find the *IS* and *LM* equations for this economy. Take total differentials of these two equations and express in matrix form, $\mathbf{Ax} = \mathbf{d}$.

b. Assume that an exogenous oil shock causes *P* to increase. Find the comparative static effects of this on the level of real output and interest rates. Illustrate your answer.

c. Now assume that the government decides to use monetary policy to offset the impact of rising prices on output (i.e. so that $dY = 0$). What is the relationship between the change in money supply and the change in prices in this case?

d. If instead the government decides to alter government spending to offset the impact of rising prices on output, show that *dG* is positive in this case. Illustrate your answers.

[15 marks]

5. Consider a factory that sells its product in two separate markets. The revenue from selling the product in market *i* is $R_i(x_i)$ where x_i is the amount of the product sold in market *i* and $R_i'(x_i) > 0, R_i''(x_i) < 0$. Good 1 is untaxed whereas good 2 is subject to a specific tax at rate t_2. The total cost of production is $C(x_1 + x_2) + t_2 x_2$ where $C'(\ ) > 0, C''(\ ) > 0$. The firm chooses its levels of sales in the two markets to maximise profits, which are given by:

$$\pi = R_1(x_1) + R_2(x_2) - C(x_1 + x_2) - t_2 x_2$$

a. Write down the first-order conditions for the optimal choices of x_1 and x_2. Compare the levels of marginal revenue in the two markets. Provide some intuition for your results.

b. Check that the second-order sufficient condition for the local profit maximum is satisfied. Can you say whether the values of x_1 and x_2 which satisfy the first-order conditions lead to a global profit maximum?

c. Find the comparative static effects of a change in t_2 on x_1^* and x_2^* and on aggregate production.

[15 marks]

6. The standard consumer problem is to maximise utility subject to a budget constraint: $\underset{x_1, x_2}{Max}\ u(x_1, x_2)\ s.t\ I = p_1 x_1 + p_2 x_2$.

a. Use a bordered Hessian to establish the circumstances under which $u(x_1, x_2)$ will be a strictly quasiconcave function.

b. Use the Lagrange method to find the first order conditions for utility maximisation. Show that the second order conditions for a local maximum are satisfisfied as long as $u(x_1, x_2)$ is a strictly quasiconcave function.

c. Take total differentials of the first order conditions and use these to find the comparative static effects of a change in *I* on x_1 and x_2. Under what conditions will x_1 be normal and x_2 be inferior?

[15 marks]

7. Consider a firm with the production function $Y = L^{1/2} + \log K$ where L is labour, K is capital and Y is output. The firm takes the price of its output, p, the price of labour, w, and the price of capital, r as exogenous.

a. Set up the profit-maximisation problem of the firm and derive the first-order conditions for a profit maximum.

b. Are the second-order sufficient conditions for a global profit maximum satisfied?

c. Derive the firm's factor demand functions.

d. According to Hotelling's Lemma, what are the effects on *profit* of changes in r and w? Confirm Hotelling's Lemma by substituting the demand functions into the profit function and differentiating as necessary.

[15 marks]

8. A firm has the production function $Y = LK^2$. For a given output, $Y = Y_0$, the firm chooses K and L to minimise the total cost of production. The wage and rental rates are w and r, both exogenously given.

a. Use the Lagrange method to find the conditional factor demand functions for K and L.

b. Find the total cost function, $C(Y_0, w, r)$, i.e., the cost of production when K and L are being chosen optimally.

c. Find the marginal cost of production. Also find the value of the multiplier, λ^*, when K and L are being chosen optimally. Show how this is related to the marginal cost of production and then relate your answer to the Envelope Theorem.

[15 marks]

9. A consumer has a utility function $u(x_1, x_2) = x_1 x_2$ and a budget constraint given by $M = p_1 x_1 + p_2 x_2$. However, she is rationed in the market for each good in that there is a maximum amount, $\overline{x}_i$ of each good, $i = 1, 2$, that she is able to buy. That is, $x_1 \le \overline{x}_1$ and $x_2 \le \overline{x}_2$. Assume that $\overline{x}_i < M / p_i$ for both goods.

a. Illustrate the consumer's problem and identify the three possible outcomes.

b. Write down the Lagrangian and the corresponding Kuhn-Tucker conditions. Assume that the second-order conditions are satisfied.

c. Consider the three possible outcomes in turn, by finding the optimal values for x_1^* and x_2^* and by establishing a relationship between the relative price of the two goods and the marginal rate of substitution in each case. Illustrate your results.

d. Use the Envelope Theorem to interpret the three Lagrange multipliers.

[15 marks]