THE AUSTRALIAN NATIONAL UNIVERSITY

Mathematics for Economists A (ECON2125)
Mathematical Techniques in Economics 1 (ECON8013/ECON4021)

Mid-Semester Examination, April 2007

READING TIME: 15 minutes
WRITING TIME: 90 minutes

Permitted materials: Non-programmable calculators

This exam will be marked out of 90. The marks for each question are indicated at the end of the question. **Answer ALL questions**.

1. Use the points $x' = 2,\ x'' = 4$ and $\lambda = 1/4$ to show that $y = f(x) = x^2$ satisfies the definition of a strictly convex function. Illustrate your answer. Then show that this definition is satisfied $\forall x', x''$ and $\forall \lambda \in (0,1)$

[15 marks]

2. Use a second-order Taylor series approximation to $y = e^{x^2}$ about $x = 0$ to estimate the value of *y* when $x = 0.025$. What is the error associated with this approximation?

[10 marks]

3. An economy produces two commodities using labour as the only factor of production. The production functions for the two goods are as follows:

$$x_1 = 20L_1^{1/2}$$

$$x_2 = L_2$$

where x_i is the quantity of good *i* produced and L_i is labour employed in producing good *i*. Assume that the total labour supply is 100 (i.e., $L_1 + L_2 = 100$).

a. Find the marginal product of labour employed in each sector.

b. By inverting the production functions find the labour requirement function (i.e. labour as a function of output) for each sector.

c. Find x_2 in terms of x_1 (by using the total labour supply equation). Calculate dx_2/dx_1 and d^2x_2/dx_1^2 and find the signs of these derivatives. Does the production possibility frontier slope down? Is it strictly concave or strictly convex? Explain. Sketch the production possibility frontier.

d. The economy is small and open and faces the world prices of p_1 and p_2 where p_i is the price of good *i*. National income is $y = p_1x_1 + p_2x_2$. Find the level of x_1 where $dy/dx_1 = 0$. Does this maximise national income? Illustrate your answer.

e. Find the comparative static effect of a change in p_1 on x_1 and x_2. Illustrate your answer.

[20 marks]

4. An issue of political debate has been whether firms enjoying monopoly power will reduce prices if taxes on their products are reduced. To analyse this issue, assume that a monopoly's profits are given by

$$\pi = [p(x) - t]x - C(x)$$

where $p(x)$ is the price that consumers pay for each unit of the firm's product, t is the tax rate and $C(x)$ is the firm's costs. Assume $p'(x) < 0,\ xp''(x) + 2p'(x) < 0$, $C'(x) > 0$ and $C''(x) > 0$.

a. Write down the first-order condition for the firm's profit-maximisation problem and check that the second-order condition is satisfied.

b. Show how x and p will change as t falls.

[15 marks]

5. Consider the linear supply and demand system:

Demand $$Q = a - bp^c$$

Supply $$Q = c + ep^p$$

$$p^c = p^p + t$$

where Q is the quantity demanded and supplied, p^c is the price paid by consumers, p^p is the price paid by consumers and t is the tax rate. Assume that a, b, c and e are all positive constants and $a > c + t$. Express this in matrix format $Ax = d$ where x is the vector of endogenous variables: $x = \begin{pmatrix} Q \\ p^c \\ p^p \end{pmatrix}$. Solve for p^c using Cramer's Rule.

[10 marks]

6. Consider the system of equations:

$$3x_1 + x_2 + x_3 = 1$$
$$x_2 - 2x_3 = 1$$
$$2x_1 - x_2 + 4x_3 = 3$$

State whether these equations have a unique solution or not. If so, find the unique solution. If not, find whether the equations are inconsistent or have infinitely-many solutions.

[10 marks]

7 Suppose that utility, $U(x, p)$ is a function of the quantity of a consumption good consumed, *x*, and of pollution, *p*. Assume $U_x > 0$ and $U_p < 0$. Also suppose that $p = g(x)$, where $g'(x) > 0$. Describe in words the difference between the partial and total derivative of *U* with respect to *x*. Which will be larger?

[5 marks]

8. Take total differentials to find $\left.\frac{dx_2}{dx_1}\right|_{dU=0}$ for the following utility function:

$$U = [0.3x_1^{-3} + 0.7x_2^{-3}]^{-1/3}$$

[5 marks]