AUSTRALIAN NATIONAL UNIVERSITY

First Semester Examination - June 2005

MATHEMATICAL TECHNIQUES IN ECONOMICS 1
AND MATHEMATICS FOR ECONOMICS A
ECON 8013 / ECON 4021/ ECON 2125

Study Period: 30 Minutes

Time Allowed: Three Hours

Permitted Materials: Nonprogrammable Calculators

Answer ALL questions. This exam is marked out of 120. Marks for individual questions are given at the end of each question.

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1. a. Simplify $\sqrt{a^5}\big/a^2$
   b. Simplify $x_1^{b-1}x_2^c\big/x_1^b x_2^{c-1}$
   c. Simplify $(x_1^{1/b}x_2^c)^{b/c}$
   d. Simplify $2\log x + 2\log y - \log z$
   e. Find dy/dx when $y = \dfrac{x^3}{x^2 - 8x}$.
   f. Find dy/dx when $y = x[f(x)]^3$
   g. Find dy/dx when $y = e^{af(x)}$
   h. Find $\partial y/\partial x_1$ when $y = \ln(x_1{}^2 / x_2{}^2)$
   i. Find dz/dx when $z = f(y(x), x)$
   j. Find f_{21} and f_{12} when $y = f(x_1, x_2, x_3) = x_1{}^{\alpha} x_2^{\beta} x_3^{\gamma}$

[10 marks]

2. Take total differentials to find $\left.\dfrac{dy}{dx}\right|_{dU=0}$ for the following utility functions:

   a. $u(x, y) = (x - a)^{1/2}(y - b)^{1/2}$
   b. $u(x, y) = ye^x$

   For the utility function in part b., find $\left.\dfrac{d^2y}{dx^2}\right|_{dU=0}$ and show that the indifference curves are strictly convex for $x, y > 0$. What does this tell you about the utility function?

[10 marks]

3. The demand for bananas is given by

$$Q_B^D = 100 - 2P_B + 0.5P_P$$

and that for pineapples is given by

$$Q_P^D = 120 - P_P + 0.75P_B$$

where P_B and P_P are the price of bananas and pineapples respectively. The respective supply functions are

$$Q_B^S = 10 + P_B + 5W_B$$

$$Q_P^S = 5 + 2P_P + 2W_P$$

where W_B and W_P are the indexes of weather conditions affecting production of bananas and pineapples respectively. Find the comparative static effects on the equilibrium price of bananas of changes in the weather condition variables. Interpret your answer.

[10 marks]

4. Consider the production function $y = x_1^{\alpha} x_2^{1/2}, \alpha > 0$. Show whether this function is concave, strictly concave or neither for strictly positive x_1 and x_2 if:

a. $\alpha < 1/2$

b. $\alpha = 1/2$

c. $\alpha > 1/2$

[10 marks]

5. Consider the following closed-economy macroeconomic model in which consumption depends not only on income but also depends positively on real money balances. The model is summarised by the *IS*, *LM* and "supply-side" equations, respectively:

$$Y = C(Y, M/P) + I(r), \quad 0 < C_Y < 1, \; C_{M/P} > 0, \quad I'(r) < 0 \qquad (1)$$

$$\frac{M}{P} = L(Y, r), \quad L_Y > 0, \quad L_r < 0 \qquad (2)$$

$$Y = f(W/P), \quad f'(W/P) < 0 \qquad (3)$$

where Y is real output, C is consumption, I is investment, r is the interest rate, M is the nominal money supply, P is the price level and W is the nominal wage rate. In this model, M and W are exogenous and Y, r and P are endogenous.

a. Take total differentials of equations (1), (2) and (3) and express in matrix form, $\mathbf{Ax} = \mathbf{d}$. Show that the determinant of **A** is positive.

b. In this economy if W were to increase while the nominal money supply stayed unchanged, can you say whether Y increases or decreases? [Hint: In part *c* you will be allowing both M and W to change. You will reduce the amount of writing you have to do if you allow for the possibility of changes in both M and W before answering this question specifically.].

c. Now suppose that at the same time as W is increased by $dW > 0$, M is increased by $dM > 0$ and the proportionate increases are the same so that $dW/W = dM/M$. Show that in this case Y does not change. Use the supply-side equation to compare the proportionate change in P with the proportionate increase in W.

[20 marks]

6. Consider a firm with the generalised production function $y = f(K, L)$

where L is labour, K is capital and y is output. The firm takes the price of its output, p, the price of labour, w, and the price of capital, r as exogenous. In addition, the firm receives a wage subsidy, which reduces the cost of each unit of labour employed by s. This means that the firm chooses its inputs of capital and labour to maximise its profits $\pi = pf(K, L) - (w - s)L - rK$.

a. Find the first-order conditions necessary for a profit maximum.

b. What are the second-order sufficient conditions (SOSCs) for a global profit maximum? What conditions must be placed on f_{KK} and f_{LL} in order for these to be satisfied?

c. Find the comparative static effects of an increase in the price of capital on the firm's optimal choice of capital, verifying that the firm's demand function for capital will slope down whenever the SOSCs are satisfied.

d. Find the comparative static effects on the firm's optimal choice of inputs of an increase in the subsidy rate. Is either of these unambiguous in sign? Explain.

e. How will the increase in s affect the firm's profits?

[20 marks]

7. Consider an individual with the utility function: $U(x_1, x_2) = x_1^{1/2} + x_2$ facing a budget constraint $I = p_1x_1 + p_2x_2$.

a. Find the Marshallian demand functions for the two goods. What conditions must be placed on I, p_1 and p_2 to ensure that x_2 will be positive? Are these demand functions homogeneous and, if so, to what degree?

b. Check that the SOSCs for a local maximum are satisfied.

c. Find the level of utility obtained when quantities of the two goods are chosen optimally, $V(p_1, p_2, I)$. Find the marginal utility of income and show how it relates to the value of the Lagrange Multiplier.

d. Use the Envelope Theorem to find $\partial V / \partial p_2$ and $\partial V / \partial I$, and hence to establish Roy's Identity for x_2. Verify the identity by using $V(p_1, p_2, I)$ to find x_2.

[20 marks]

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8. An individual has 10 hours to spend at a holiday resort. This can be spent diving or playing golf. The individual has \$12 to spend. The cost of an hour of diving is \$2 and the cost of an hour of golf is \$1. Let x_1 be hours spent diving and x_2 be hours spent playing golf. The individual's utility is given by

$$U = 2\ln x_1 + \ln x_2$$

The individual's optimisation problem is to choose x_1 and x_2 to maximise utility subject to $x_1 + x_2 \leq 10$ and $2x_1 + x_2 \leq 12$.

a. Write down the Lagrangian and the corresponding Kuhn-Tucker conditions. Assume that the second-order conditions are satisfied.

b. Sketch the two constraints and identify and illustrate the five possible optimal solutions.

c. What is the optimal amount of time for the individual to spend in each of the two activities? Verify your result by working through all possibilities.

[20 marks]