THE AUSTRALIAN NATIONAL UNIVERSITY

Mathematics for Economists A (ECON2125)
Mathematical Techniques in Economics 1 (ECON8013/ECON4021)

Mid-Semester Examination, April 2005

READING TIME: 15 minutes
WRITING TIME: 90 minutes

Permitted materials: Non-programmable calculators

This exam will be marked out of 90. The marks for each question are indicated at the end of the question. **Answer ALL questions**.

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1. Provide a formal definition of strict concavity and then use the points $x' = 1$, $x'' = 2$ and $\lambda = 1/4$ to demonstrate that the function $y = \ln x$ is strictly concave. Illustrate your answer.

[10 marks]

2. Consider the function $f(x) = 1/(x-1)$.
   a. Find $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$, $\lim_{x\to1^-} f(x)$ and $\lim_{x\to1^+} f(x)$. Does this function have a limit at $x = 1$? Is it continuous at $x = 1$?
   b. Find $f'(x)$ and $f''(x)$ and sketch the function.

[10 marks]

3. Suppose the inverse demand function facing a monopolist is

$$p = q^{-1/\varepsilon}$$

where p is the price of the good and q is its quantity. This implies that the elasticity of demand for this good is constant and equal to ε, assumed to be positive. Assume that the total cost is linear in quantity, given by $TC = cq$.

   a. Set up the firm's profit-maximisation problem and first-order condition in order to solve for the optimal level of output, q^*. What conditions must be placed on ε in order for the second-order condition to be satisfied?
   b. Find the optimal price, p^*. How does this vary with ε (i.e. find $dp^*/d\varepsilon$)?

[15 marks]

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4. Consider an industry with two firms, *A* and *B*. Each firm's labour demand depends on the wage, *W*, and the overall demand for goods, *Y*. Assume that the responsiveness of labour demand to overall demand (for goods) differs across the two firms, such that:

$$L_A = -\gamma W + \beta_A Y \qquad (1)$$

$$L_B = -\gamma W + \beta_B Y \qquad (2)$$

The two firms draw on a common pool of labour $\overline{L}$ and consequently pay the same wage. The number of workers in the common pool of labour is fixed and equal to $\overline{L}$ and labour markets are assumed to clear, so that:

$$\overline{L} = L_A + L_B \qquad (3)$$

Hence, L_A, L_B and W are endogenous variables, Y and $\overline{L}$ are exogenous variables and γ, β_A, β_B are fixed constants.

a. Take total differentials of equations (1), (2) and (3).
b. Use Cramer's Rule to determine the effect on L_A and L_B when there is an increase in overall demand, *Y*. Under what conditions will L_A increase and L_B fall?
c. Use Cramer's Rule to determine the effect on wages when there is an increase in the available pool of labour.
d. Confirm your answer in c. by substituting equations (1) and (2) into equation (3) and taking the total differential of the resulting single equation.

[20 marks]

5. For what values of *a* and *b* will the following system of equations have a unique solution, infinite solutions or no solutions?

$$x_1 + 2x_2 + 3x_3 = 1$$

$$-x_1 + ax_2 - 21x_3 = 2$$

$$3x_1 + 7x_2 + ax_3 = b$$

[20 marks]

6. Use implicit differentiation to find dy/dx when $e^{x^2+y} - 5 = 0$. Then take the total differential and use the Implicit Function Rule to confirm your answer.

[7 marks]

7. Suppose that both the amount of capital at time $t, K = K(t)$, and the efficiency with which it is used over time affect the level of output, *Y*, according to the function:

$$Y = f(K,t) = 0.2(1+t)^{1/2}K$$

$$\text{where } K(t) = K_0 e^{0.05t}$$

Find and give the economic tuition of the partial and total derivative of *Y* with respect to *t*.

[8 marks]