Australian National University

Mathematics for Economists A (ECON2125)
Mathematical Techniques in Economics 1 (ECON8013/ECON4021/IDEC8015)

Mid-Semester Examination, May 2004

You have **15 minutes** of reading time and **90 minutes** of writing time for this exam. It will be marked out of 100. The marks for each question are indicated at the end of the question. **Answer ALL questions**.

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1. The utility set of a consumer is given by $U = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 x_2 > \overline{u}\}$. The consumer's budget set is given by $B = \{(x_1, x_2) \in \mathbb{R}_+^2 : p_1 x_1 + p_2 x_2 \leq M\}$.

   a. Sketch the level set $L = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 x_2 = \overline{u}\}$.

   b. Illustrate the set U and the set B for the case where $X = U \cap B \neq \varnothing$.

   c. Explain whether the set X is closed? Open? Bounded? Compact? Convex? Strictly convex?

[15 marks]

2. Consider the function $y = e^{-x^2}, x \geq 0$.

   a. Find $\lim_{x \to \infty} e^{-x^2}$ and $\lim_{x \to 0} e^{-x^2}$.

   b. Find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$. For what values of x will the function be strictly convex? For what values of x will the function be strictly concave?

   c. Use the information in a. and b. to sketch the function.

[15 marks]

3. Suppose that the matrix $\mathbf{P} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$ describes population movements between two regions, where p_{ij} is the proportion of the population in region j that moves to region i. The population (in millions) of the two regions at time $t = 0$ is given by $\mathbf{x}^0 = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$.

   a. Show that the eigenvalues of $\mathbf{P}$ are $\lambda_1 = 1/3,\ \lambda_2 = 1$.

   b. Find the corresponding eigenvectors and form the orthogonal matrix **Q.**

   c. Given that population at time t is given by $\mathbf{x}^t = \mathbf{P}^t \mathbf{x}^0$, and using the fact that $\mathbf{P}^t = \mathbf{Q} \Lambda^t \mathbf{Q}^{-1}$, where $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, find the population at time $t = 2$.

[20 marks]

4. Suppose that a firm produces output, Y, using labour, L, as its only factor of production. The production function is $Y = f(L)$, where $f'(L) > 0$ and $f''(L) < 0$. The firm is a monopsonist, setting a wage rate that increases with employment such that $w = aL,\ a > 0$. In addition to paying labour the wage, w, the firm must contribute cw ($0 < c < 1$) to an employee pension scheme, so the firm pays $w(1+c)$ per unit of labour. Sales of output are subsidised at a flat rate, s, so the firm receives $(p+s)$ per unit of output.

a. Write down the firm's profit maximisation problem.
b. Find the first-order necessary condition and interpret.
c. Is the second-order condition satisfied for a local maximum level of L? For a global maximum?
d. Find the comparative static effects of a change in s on the optimal choice of labour input. Does an increase in the subsidy increase or decrease labour input? Repeat your analysis for a change in c.

[20 marks]

5. Consider a market in which consumer demand is partly met through domestic sector production and partly met through imports. All sales are subject to a tax of $\$t$ per unit so consumers pay a price per unit of $p+t$. The demand, supply and market equilibrium conditions are respectively:

$$q^d = a - b(p+t) \qquad (1)$$

$$q^s = c + ep \qquad (2)$$

$$q^d = q^s + M \qquad (3)$$

where q^d denotes quantity demanded, q^s denotes quantity supplied by the domestic sector and M denotes imports. In this model a, b, c and e are fixed constants, t and M are exogenous and q^d, q^s and p are endogenous.

a. Take total differentials of the three equations above.
b. Use Cramer's Rule to find the effect on p from an increase in M of dM. Also find the effect on p from an increase in t of dt. In each case find the sign of the change in p.
c. Confirm your answers in (b) by substituting equations (1) and (2) into equation (3) and taking the total differential of the resulting single equation.

[20 marks]

6. Consider the matrix $\mathbf{A} = \begin{bmatrix} a & 0 & 0 \\ 1 & b & 1 \\ 0 & 0 & c \end{bmatrix}$. For what values of a, b and c will $\mathbf{A}$ be positive definite? Negative definite?

[10 marks]